4.2.3 Independence of Axioms in Projective Geometry

It is common sense to take a method and try it; if it fails, admit it frankly and try another. But above all, try something.
—[Franklin D. Roosevelt](#) *(1882–1945)*

Axiom 1. Any two distinct points are incident with exactly one line.

Axiom 2. Any two distinct lines are incident with at least one point.

Axiom 3. There exist at least four points, no three of which are collinear.

Exercise 4.7. Consider just the first three axioms. Show they are independent. *(Similar to problems in Chapter One. [Examples of Axiomatic Systems](#).)*

Axiom 4. The three diagonal points of a complete quadrangle are never collinear.

Exercise 4.8. Show Axiom 4 is independent of Axioms 1–3. *(Hint. Consider the projective plane of order 2 in Chapter One. [A Finite Plane Projective Geometry](#).)*

Axiom 5. (Desargues' Theorem) If two triangles are perspective from a point, then they are perspective from a line.

Challenge Exercise 4.9. Show Axiom 5 (Desargues' Theorem) is independent of Axioms 1–4.

Axiom 6. If a projectivity on a pencil of points leaves three distinct points of the pencil invariant, it leaves every point of the pencil invariant.

Challenge Exercise 4.10. Show Axiom 6 is independent of Axioms 1–5.

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